Gluon Vortices and Induced Magnetic Field in Compact Stars

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Abstract. The natural candidates for the realization of color superconductivity are the extremely dense cores of compact stars, many of which have very large magnetic fields, especially the so-called magnetars. In this paper we discuss how a color superconducting core can serve to generate and enhance the stellar magnetic field without appealing to a magnetohydrodynamic dynamo mechanism.

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INTRODUCTION

Neutron stars created in the aftermath of supernova explosions are so compact that gravity, the weakest force in nature, binds nucleons there 10 times more strongly than the strong-nuclear force binds nucleons in nuclei. At the tremendous densities reached in the cores of those compact objects one expects the realization of a quark deconfined phase that produces a color superconducting state [1, 2].

A common characteristic of neutron stars is their strong magnetization. Their surface magnetic fields range from $B = 1.7 \times 10^8 G$ (PSR B1957+20) up to $2.1 \times 10^{13} G$ (PSR B0154+61), with a typical value of $10^{12} G$ [3]. There are observational discoveries of even strongly magnetized stellar objects- known as magnetars- with surface magnetic fields of order $B \sim 10^{14} - 10^{16} G$ [4]. In the core of these compact objects, the field may be considerably larger due to flux conservation during the core collapse. By applying the scalar virial theorem it can be shown that the interior field can reach values of order $B \sim 10^{18} G$ [5].

The observed stellar magnetic fields are supposed to be created by a magnetohydrodynamic dynamo mechanism that amplifies a seed magnetic field due to a rapidly rotating protoneutron star. Thus, the standard explanation of the origin of the magnetar's large magnetic field implies that the rotation should have a spin period $< 3ms$. Nevertheless, this mechanism cannot explain all the features of the supernova remnants surrounding these objects [6, 7]. Part of this rotational energy is supposed to power the supernova through rapid magnetic braking, implying that the supernova remnants associated with magnetars should be an order of magnitude more energetic than typical supernova remnants. However, recent calculations [6] indicate that their energies are similar. In addition, one would expect that when a magnetar spins down, the rotational energy output should go into a magnetized particle wind of ultrarelativistic electrons and positrons that radiate via synchrotron emission across the electromagnetic spectrum. Nevertheless, so far nobody has detected the expected luminous pulsar wind nebulae around magnetars

[8]. On the other hand, some magnetars emit repeated flares or bursts of energy in the range of $10^{42} - 10^{46} erg$ [9]. Since the emitted energy significantly exceeds the energy loss in their rotational energy in the same period, it is natural to expect that the energy unbalance could be supplied by the stellar magnetic field, which is the only known additional energy source. However, from the spin history of these objects, there is no clear evidence of the surface magnetic field damping [10].

Although more observations are needed to confirm the above issues, current data indicate that alternative models to the standard magnetar model [4] need to be considered. For example, some authors [6, 11] have suggested that magnetars could be the outcome of stellar progenitors with highly magnetized cores. In this paper I want to show that a progenitor star with a color superconducting core would be capable of inducing and/or enhancing its magnetic field through mechanisms different than those of the standard magnetar [4].

The color superconducting state of dense quark matter exhibits a rich spectrum of different phases. For matter densities high enough to guarantee the quark deconfinement, and sufficiently low to consider the decoupling of the strange quark mass, M_s, the quarks can condense to form a two-flavor color superconductor (2SC) [1]. At moderately higher densities, for which M_s cannot be regarded extremely large, the three-flavor phases, $gCFL$ [12] and CFL [2], will appear respectively with increasing densities. Matter in the stellar regions should be neutral and remain in β equilibrium. When these conditions are taken into account in the $2SC$ phase [13], or when put together with the M_s contribution into the $gCFL$ phase [14], some gluon modes become tachyonic, indicating that the system ground state should be restructured to remove the created chromomagnetic instability. Several interesting proposals have already been worked out to find the stable ground state at the realistic, moderate densities of compact stars. Some of the most promising possibilities are a modified CFL-phase with a condensate of kaons [15] that requires a space-dependent condensate to remove the instability, a LOFF phase in which the quarks pair with nonzero total momentum [16], and homogenous [17], as well as inhomogeneous [18], gluon condensate phases. At present it is not clear if any of these proposals is the final solution to the problem; albeit, finding the ground state that minimizes the system energy at densities of interest for astrophysical applications is a pressing problem which is attracting much attention.

In the following, I will discuss the mechanisms that can enhance [19] and/or induce [18] magnetic fields in color superconducting states. The realization of such mechanisms in the core of compact stars can be an interesting alternative in order to address the main criticisms pointed out above to the standard magnetar's paradigm. On the other hand, having a mechanism that associates the existence of high magnetic fields with color superconductivity at moderate densities can serve to single out the magnetars as the most probable astronomical objects for the realization of this high-dense state of matter.

MAGNETIC-FIELD INCREASE BY GLUON VORTICES IN COLOR SUPERCONDUCTORS

The interaction of an external magnetic field with dense quark matter has been investigated by several authors [18] - [22]. Here I want to consider the effects of an external

magnetic field in the gluon dynamics of a color superconductor with three massless quark flavors (*CFL* phase) [19], but the same effect can be produced in the two-flavor theory (2*SC* phase). The ground state of the *CFL* phase is formed by spin-zero Cooper pairs in the color-antitriplet, flavor-antitriplet representation. An important feature of spin-zero color superconductivity is that, although the color condensate has non-zero electric charge, the linear combination $\widetilde{A}_\mu = \cos\theta\, A_\mu + \sin\theta\, G^8_\mu$ of the photon A_μ and the gluon G^8_μ remains massless [20, 23], so it behaves as an in-medium electromagnetic field, while the orthogonal combination $\widetilde{G}^8_\mu = -\sin\theta A_\mu + \cos\theta\, G^8_\mu$ is massive. In the CFL phase the mixing angle θ is small, thus the penetrating "electromagnetic" field is mostly formed by the original photon with only a small gluon admixture. Even though gluons are neutral with respect to the conventional electromagnetism, in the *CFL* phase they are charged with respect to the rotated field acquiring $\widetilde{Q}$ charges:

G^1_μ	G^2_μ	G^3_μ	G^+_μ	G^-_μ	I^+_μ	I^-_μ	$\widetilde{G}^8_\mu$
0	0	0	1	-1	1	-1	0

, (1)

given in units of $\widetilde{e} = e\cos\theta$. The $\widetilde{Q}$-charged fields in (1) correspond to the combinations $G^\pm_\mu \equiv \frac{1}{\sqrt{2}}[G^4_\mu \mp iG^5_\mu]$ and $I^\pm_\mu \equiv \frac{1}{\sqrt{2}}[G^6_\mu \mp iG^7_\mu]$. The external rotated magnetic field $\widetilde{H}$ that penetrates the CFL phase produces an instability when its strength becomes larger than the Meissner mass m_M of the $\widetilde{Q}$-charged gluons. The tachyonic mode can be easily found by diagonalizing the mass matrix of the field components $(G^\pm_1, G^\pm_2)$

$$\begin{pmatrix} m_M^2 & i\widetilde{e}\widetilde{H} \\ -i\widetilde{e}\widetilde{H} & m_M^2 \end{pmatrix} \rightarrow \begin{pmatrix} m_M^2 + \widetilde{e}\widetilde{H} & 0 \\ 0 & m_M^2 - \widetilde{e}\widetilde{H} \end{pmatrix} \tag{2}$$

Clearly, above the critical field $\widetilde{e}\widetilde{H}_C = m_M^2$ the lowest mass mode in (2) becomes tachyonic, with corresponding eigenvector of amplitude G in the $(1,i)$ direction for G^- (G^* in the $(1,-i)$ direction for G^+). This phenomenon is the well known "zero-mode problem" found in the presence of a magnetic field for Yang-Mills fields [24], for the $W^\pm_\mu$ bosons in the electroweak theory [25, 26], and even for higher-spin fields in the context of string theory [27]. To remove the tachyonic mode, the ground state is restructured through the formation of a gluon condensate of amplitude G, as well as an induced magnetic field $\widetilde{B}$ that originates due to the backreaction of the G condensate on the rotated electromagnetic field. The condensate solutions can be found by minimizing with respect to G and $\widetilde{B}$ the Gibbs free-energy density $\mathscr{G}_c = \mathscr{F} - \widetilde{H}\widetilde{B}$, ($\mathscr{F}$ is the free energy density)

$$\mathscr{G}_c = \mathscr{F}_{n0} - 2G^\dagger \widetilde{\Pi}^2 G - 2(2\widetilde{e}\widetilde{B} - m_M^2)|G|^2 + 2g^2|G|^4 + \frac{1}{2}\widetilde{B}^2 - \widetilde{H}\widetilde{B} \tag{3}$$

In (3) $\mathscr{F}_{n0}$ is the system free energy density in the normal-CFL phase ($G=0$) at zero applied field. Using (3) the minimum equations at $\widetilde{H} \sim \widetilde{H}_C$ for the condensate G and induced field $\widetilde{B}$ respectively are

$$-\widetilde{\Pi}^2 G - (2\widetilde{e}\widetilde{B} - m_M^2)G = 0, \tag{4}$$

$$2\widetilde{e}|G|^2-\widetilde{B}+\widetilde{H}=0 \tag{5}$$

Identifying G with the complex order parameter, Eqs. (4)-(5) become analogous to the Ginzburg-Landau equations for a conventional superconductor except for the $\widetilde{B}$ contribution in the second term in (4) and the sign of the first term in (5). The origin of both terms can be traced back to the anomalous magnetic moment term $4\widetilde{e}\widetilde{B}|G|^2$ in the Gibbs free energy density (3). Notice that because of the different sign in the first term of (5), contrary to what occurs in conventional superconductivity, the resultant field $\widetilde{B}$ is stronger than the applied field $\widetilde{H}$. Thus, when a gluon condensate develops, the magnetic field will be antiscreened and the color superconductor will behave as a paramagnet. The antiscreening of a magnetic field has been also found in the context of the electroweak theory for magnetic fields $H\geq M_W^2/e\sim 10^{24}G$ [26]. Just as in the electroweak case, the antiscreening in the color superconductor is a direct consequence of the asymptotic freedom of the underlying theory [26].

The explicit solution of (4)-(5) can be found following Abrikosov's approach [28] to type II metal superconductivity for the limit situation when the applied field is near the critical value H_{c2}. In our case one finds [19] that the ground state is restructured, forming a vortex state characterized by the condensation of charged gluons and the creation of magnetic flux tubes. In the vortex state the magnetic field outside the flux tubes is equal to the applied one, while inside the tubes its strength increases by an amount that depends on the amplitude of the gluon condensate. This non-linear paramagnetic behavior of the color superconductor can be relevant to boosting the star's magnetic field without appealing to a magnetohydrodynamic dynamo mechanism.

INDUCED MAGNETIC FIELD IN A COLOR SUPERCONDUCTOR AT MODERATE DENSITIES

Let us consider now the Meissner unstable region of the so-called gapped 2*SC* phase [29] (we expect that the obtained results can be also realized in the Meissner unstable region of the three-flavor theory). As is known [13], the gapped 2SC turned out to be unstable for certain values of the baryonic chemical potential, even at zero applied magnetic field, once the gauge fields $\{G_\mu^{(1)},G_\mu^{(2)},G_\mu^{(3)},K_\mu,K_\mu^\dagger,\widetilde{G}_\mu^8,\widetilde{A}_\mu\}$ were taken into consideration. Note that even though the gluons are neutral with respect to the regular electric charge, in the 2SC phase the gluon complex doublet K_μ has nonzero rotated charge of magnitude $\widetilde{q}=\widetilde{e}/2$, with $\widetilde{e}=e\cos\theta$. The tachyonic modes in this unstable phase are associated only with those charged gluon fields. It is natural to expect then that a new, stable ground state should incorporate the condensation of the K_μ gluons. In general we should allow for an inhomogeneous condensate, and taking into account that this kind of solution may generate rotated electromagnetic currents, the rotated magnetic field should be included in the general framework of the condensation phenomenon, but now as an induced field created in the absence of an applied field.

Borrowing from the experience gained in the case with external magnetic field, we propose the following ansatz for the condensate solution

$$\langle K_\mu^T\rangle\equiv(\overline{G}_\mu,0)/\sqrt{2},\quad \overline{G}_\mu\equiv\overline{G}(x,y)(0,1,-i,0), \tag{6}$$

where we take advantage of the $SU(2)_c$ symmetry to write the $\langle K_i \rangle$-doublet with only one nonzero component. Since in this ansatz the inhomogeneity of the gluon condensate is taken in the (x,y)-plane, it follows that the corresponding induced magnetic field will be aligned in the perpendicular direction, i.e. along the z-axes, $\langle \widetilde{f}_{12} \rangle = \widetilde{B}$. The part of the free energy density that depends on the gauge-field condensates is given by

$$\mathscr{F}_g = \frac{\widetilde{B}^2}{2} - 2\overline{G}^* \widetilde{\Pi}^2 \overline{G} + 2g^2 |\overline{G}|^4 - 2[2\widetilde{q}\widetilde{B} + (\mu_8 - \mu_3)^2 - m_M^2] |\overline{G}|^2 \tag{7}$$

where the "chemical potentials" [30] μ_8 and μ_3 are condensates of the time components of gauge fields, $\mu_8 = (\sqrt{3}g/2)\langle G_0^{(8)} \rangle$ and $\mu_3 = (g/2)\langle G_0^{(3)} \rangle$. From the neutrality condition for the 3^{rd}-color charge it is found that $\mu_3 = \mu_8$. The fact that μ_3 gets a finite value just after the critical point $m_M^2 - \mu_8^2 = 0$ is an indication of a first-order phase transition, but since μ_8 is parametrically suppressed in the gapped phase by the quark chemical potential [13], it will be a weakly first-order phase transition. As follows, we will consider that $\mu_3 = \mu_8$ in (7), and work close to the transition point $\delta\mu_c$, which is the point where m_M^2 continuously changes sign to a negative value. In that region, the minimum equations for the condensate G^* and induced field $\widetilde{B}$ are respectively given by [18]

$$\widetilde{\Pi}^2 \overline{G} + \widetilde{q}\widetilde{B}\overline{G} \simeq 0. \tag{8}$$

$$2\widetilde{q}|\overline{G}|^2 - \widetilde{B} \simeq 0 \tag{9}$$

The relative sign between the two terms in Eq. (9) implies that for $|\overline{G}| \neq 0$ a magnetic field $\widetilde{B}$ is induced. This effect has the same physical root as the paramagnetism found in the last section. The explicit form of the inhomogeneity can be found from (8) in the domain $r \ll \xi$ (where $\xi^2 \equiv (g^2/\widetilde{q}^2 - 1)/|m_M^2|$ is the characteristic length) as [18]

$$|\overline{G}|^2 \simeq \frac{1}{2\widetilde{q}^2 \xi^2} - \frac{r^2}{4\widetilde{q}^2 \xi^4} \tag{10}$$

The solution for $\widetilde{B}$ is found substituting (10) back into (5). The induced field $\widetilde{B}$ is homogeneous in the z-direction and inhomogeneous in the (x,y)-plane.

Finally, it should be highlighted that when the absolute value of the magnetic mass becomes of order m_g [13], the gluon condensate could produce a magnetic field $\sim 10^{16} - 10^{17} G$. The possibility of generating a magnetic field of such a large magnitude in the core of a compact star without relying on a magneto-hydrodynamic effect can be an interesting alternative to address the main criticism [6] of the observational conundrum of the standard magnetar paradigm. Then, one concludes that if color superconductivity is realized in the core of compact stellar objects at such expected densities that a Meissner unstable phase is attained, the theory of the origin of stellar magnetization should consider the discussed mechanisms.

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